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Longitudinal vortices in natural convection flow on inclined plates

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Experiments are performed to demonstrate the occurrence and explore the characteristics of a secondary flow superposed upon the natural convection main flow on an inclined plate. A flow visualization technique is employed whereby the flow pattern is made visible by local changes of colour of the fluid itself, the colour change being brought about by a change in pH. The secondary flow consists of longitudinal vortices or rolls distributed more or less periodically across the width of the plate. The number of such vortices increases with the temperature difference between the surface and the ambient fluid, but appears to be relatively insensitive to the inclination angle of the plate. The secondary flow results from the destabilizing effect of the buoyancy force component, which acts normal to the plate surface. The longitudinal vortices are the first stage of the laminar-turbulent transition process. This is in contrast to the case of natural convection on a vertical plate, where the first stage of transition is Tollmien–Schlichting waves.

Introduction

The occurrence of longitudinal vortices or rolls in forced convection boundary-layer flows on concave walls is well established. As was first shown by Görtler (1954) and further elucidated by subsequent investigators, the vortices arise from the destablizing effect of the centrifugal forces associated with the concavity of the wall. In later papers, Görtler (1959) and Kirchgässner (1962) demonstrated analytically that a buoyancy force component directed normal to a wall would have a similar effect on a forced convection main flow. Longitudinal vortices were also observed by Chandra (1938) in a horizontal channel, the lower bounding surface of which was heated while the upper surface was in longitudinal motion.

In the present research, the occurrence of longitudinal vortices in natural convection flow on inclined plates is investigated, such vortices not having been previously observed in this flow configuration.

Experiments pertaining to natural convection on an inclined plate are reported by a number of investigators. Interferometer and shadowgraph methods were used by Kierkus (1968), Rich (1953) and Schmidt (1932) in determining natural convection heat transfer characteristics of inclined plates. These measurement techniques tend to average along the direction of the optical path (i.e. across the width of the plate), thereby precluding the observation of three-dimensional phenomena such as longitudinal vortices. Tritton (1963) employed a fibre anemometer to detect laminar-turbulent transition, but restricted his observations to the mid-point of the width. The velocity field measurements of Kierkus (1968), obtained from dust particle trajectories, were also presumably confined to a specific spanwise position.

The flow visualization technique employed herein is one in which the fluid motions are made visible by local changes of colour of the fluid itself, the colour changes resulting from changes in pH (acidic to basic). By arranging for the colour change to occur either at the plate surface or within the boundary layer, various aspects of the flow field may be examined. In particular, the three-dimensional character of the flow, with longitudinal vortices superposed on the basic natural convection motion, is clearly indicated.

Experimental data, primarily in the form of flow field photographs, will be presented and discussed following the description of the experimental apparatus and the flow visualization technique.

Apparatus and measurements

The heated plate assembly used in these experiments was fabricated from aluminium. The interior of the plate consisted of fourteen longitudinal channels, through which was passed a fluid from a constant temperature bath. The external surfaces of the plate assembly were plated with nickel. Thin strips of plexiglass were affixed to the lateral edges to minimize extraneous disturbances. The dimensions of the test surface were 21·5 cm (length) by 20 cm (width).

The plate assembly was positioned at one end of a glass-walled tank filled with a solution consisting primarily of water (the other constituents will be described subsequently). The tank dimensions were $58 \times 30 \times 40$ cm, length by width by height. The lower edge of the plate rested on a plastic support such that the leading edge of the test surface was about 5 cm from the floor of the tank. The inclination angle of the plate was adjusted manually, the angle being read with the aid of a protractor. The upward-facing surface of the inclined plate constituted the test surface.

The flow visualization technique was adapted from that described by Baker (1966). Thymol blue, a pH indicator, is added to the water in an amount approximately 0·01 % by weight. (Thymol blue is blue in colour when indicating a basic solution and yellow-orange in colour when indicating an acidic solution.) The solution is titrated to the end point (pH $\sim$ 8) with sodium hydroxide. Then, by the addition of hydrochloric acid, the solution is made acidic and yellow-orange in colour. When a small d.c. voltage ($\sim$ 10–20 volts) from a dry-cell source is impressed between two electrodes situated within such a fluid, there is a proton transfer reaction at the negative electrode. As a result, there is a change of pH of the fluid (acid to base) at the surface of the negative electrode, with a corresponding change in colour from yellow-orange to blue. The thus-created dye is neutrally buoyant and faithfully follows the motion of the fluid.

In the present experiments, two electrode arrangements were employed. In the first arrangement, the test surface itself served as the negative electrode, so that the tracer fluid was generated right at the surface. The positive electrode was a pair of copper sheets situated adjacent to the side walls of the tank, well removed from the test surface. For the second arrangement, the negative electrode was a tungsten wire of 0·076 mm (0·003 in.) diameter and 17 cm length positioned in the boundary layer, parallel to the plate surface and transverse to the direction of the free convection main flow. The distance between the wire and the surface could be set arbitrarily, as could the longitudinal location of the wire relative to the leading edge. The test surface served as the positive electrode. With the wire as negative electrode, the spanwise distribution of up-flow and down-flow zones in the boundary layer could be observed.

The flow patterns thus made visible were recorded photographically. Two camera arrangements were employed in order to obtain a range of information. The first (and simpler) arrangement was one in which the camera looked head-on at the test surface. In the second arrangement, a prism was utilized so as to view along a line of sight parallel to the plate surface and looking upward along the plate from the leading edge. Such would be the view from a point of observation situated on the floor of the tank. The prism was, indeed, positioned on the floor of the tank, and the camera, looking downward into the prism, viewed the scene looking upward along the plate from the leading edge.

The first of the just-mentioned camera arrangements was utilized in the experiments in which the test surface was the negative electrode, while the second camera arrangement was employed when either the test surface or the wire served as the negative electrode. The photographs were taken on Tri-x film with an Exakta camera, with lighting provided by a diffuse fluorescent source. Prints were made on F 6 contrast grade paper.

The constant temperature bath utilized for heating and circulating water through the test apparatus was a Lauda model NBS, capable of controlling the temperature to within $\pm 0{\cdot}01$ to $\pm 0{\cdot}03\,°C$. Three copper constantan thermocouples were imbedded in the test surface. The temperature of the fluid within the tank was measured with laboratory grade thermometers.

Experimental results

The first experiments to be discussed are those in which the plate served as the negative electrode, so that the tracer fluid was generated right at the test surface itself. With this arrangement, and if the flow were two-dimensional and laminar (i.e. no longitudinal vortices), the plate would be blanketed by a film of blue fluid moving parallel to the surface. Aside from this gross upward motion, no other details of the flow field would be discernible. The experimental observations are very different from the aforementioned visualization pattern for a two-dimensional flow, as is demonstrated by the photographs of figure 1, plate 1. (Owing to the lighting arrangement, the lower portion of the plate is in darkness at smaller angles of inclination α.) The angle α appearing in the figure is measured relative to the vertical; ΔT is the temperature difference between the plate and the fluid.

It is seen by inspection of the figure that there is an array of more or less regularly spaced lines of tracer fluid arranged across the width of the plate. Since the tracer fluid is generated continuously all along the plate surface, the presence of the discrete lines is indicative of spanwise motions of a cellular nature. As will be affirmed later by additional photographic information, the lines of tracer fluid correspond to flow normal to the test surface, in the direction away from the surface. Indeed, each line is the outflow leg of a cell of secondary flow (i.e. of a longitudinal vortex). The region between a pair of lines contains the inflow leg of such a cell. The absence of tracer fluid in an inflow leg results from a spanwise (i.e. sidewise) motion near the plate surface which sweeps fluid toward the outflow leg.

The presence of lines such as those of figure 1 was discernible for inclination angles α of 15° and greater. The phenomena of cell merging and subsequent destruction, evidenced in figure 1 for $\alpha = 35°$, was also characteristic of larger angles of inclination. The effect of increasing α was to translate the entire pattern of lines downward toward the leading edge. The effect of inclination angle and temperature difference on cell size will be discussed later.

Next, attention will be turned to flow field information obtained by looking upward along the plate from the leading edge, the plate serving as the negative electrode and as the source of the tracer fluid. Figure 2 is illustrative of the flow field observations obtained under these conditions. The plate surface coincides with a horizontal line drawn through the lowest points of the white region of the figure. The black regions situated above the aforementioned line correspond to the tracer fluid, while the white regions are the remainder of the flow field.

Figure 2 clearly indicates a periodic cell pattern distributed across the span of the plate. The tracer fluid, generated at the plate surface, is at first swept sideways (note curvature of the bottom portions of the white regions). The merging of the sidewise flows from two adjacent cells results in an outflow of the tracer fluid, normal to the plate.

The photographic view shown in figure 2, plate 2, is an integration of flow field observations along a line of sight parallel to the plate surface, looking upward from the leading edge. The uppermost portions of the outflowing streams of tracer fluid that are seen in figure 2 are actually located at downstream positions. That is, the outflowing stream moves farther and farther away from the plate surface with increasing downstream distance.

Now flow field observations with the tungsten wire as negative electrode will be presented and discussed. As noted earlier, the wire is situated in the boundary layer, parallel to the plate and transverse to the direction of the natural convection main flow. With the aid of the prism, the wire is viewed along a line of sight parallel to the plate surface, looking upward from the leading edge.

If the natural convection flow field were two-dimensional, the tracer fluid generated by the wire would, when viewed in the manner just discussed, appear as a uniform sheet, flowing from the wire toward the plate. This is because the transverse velocity in a conventional natural convection boundary layer (i.e. the boundary layer on a vertical plate) is directed toward the plate surface. The actual flow pattern adjacent to the wire is altogether different from that for a two-dimensional flow, as may be seen from the representative set of photographs presented in figure 3. The three photographs correspond to the same temperature difference, same angle of inclination, same distance of the wire from the surface (2·5 mm), but to three different streamwise locations, $x = 4{\cdot}3$, 6·5, and 8·0 cm from the leading edge. (Any apparent differences in the distance of the wire from the surface are due to slight differences in the direction of viewing.)

Figure 3, plate 3, shows a cell pattern that is consistent with the prior observations and discussion. In the upper two photographs, tongues of inward flowing fluid, periodically spaced along the wire, are in evidence. Between these tongues, the inflow is suppressed by an outflow which, itself, has not reached the location of the wire. In the lower photo, successive inflow and outflow zones are clearly in evidence.

The effect of the temperature difference ΔT and the inclination angle α on the cell size will now be discussed. It is found that the cell size is strongly affected by the magnitude of ΔT, decreasing as ΔT increases. For instance, at a fixed plate inclination of 35°, the number of lines observed in photographs such as those in figure 1 increased from 30 to 42 as ΔT was increased from 9 to 28 °C.

On the other hand, cell size appeared to be relatively weakly influenced by the angle of inclination. Tests performed for a fixed temperature difference $\Delta T = 11$ °C revealed essentially no change in the number of lines as α was varied from 30 to 60°.

Concluding remarks

The present experiments on inclined plates reveal the presence of cellular secondary flows superposed upon the natural convection main flow. It is believed that these longitudinal vortices are the first stage of the laminar-turbulent transition process. This is in contrast to natural convection on a vertical plate, where Tollmien–Schlichting waves constitute the first stage of transition.

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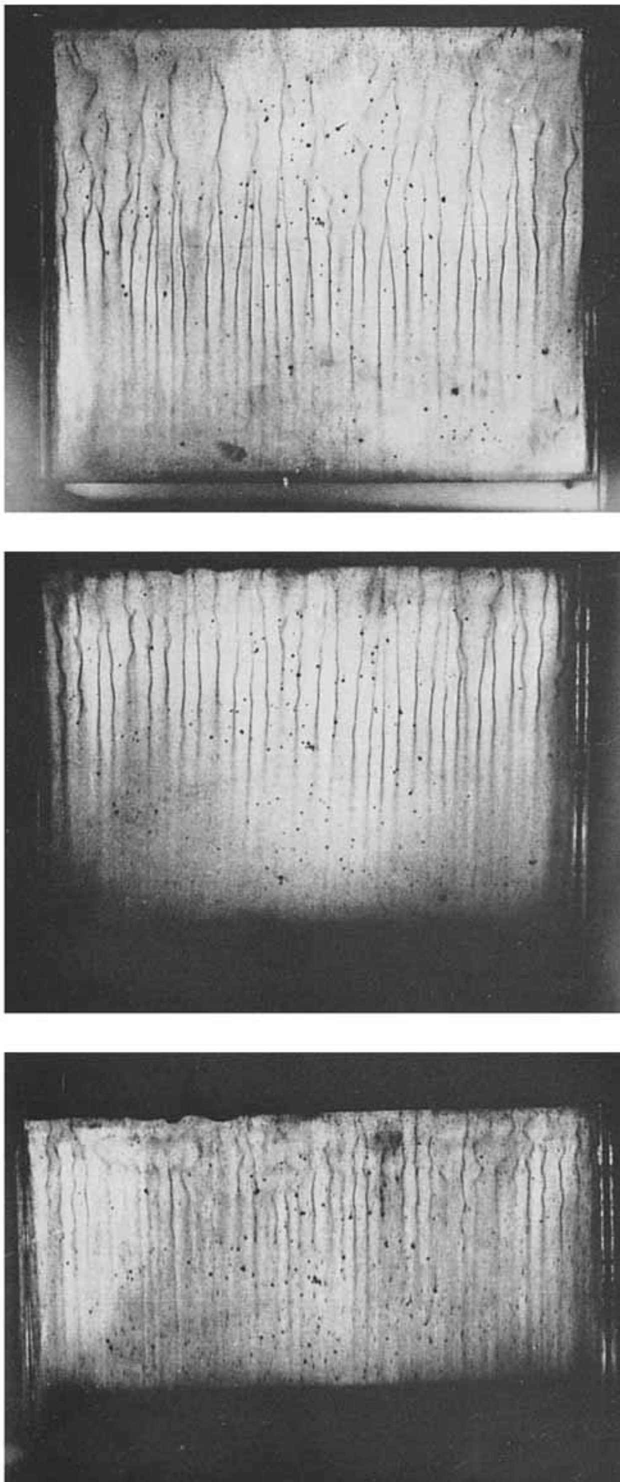


$\alpha = 35°$
$\Delta T = 11°$ C

$\alpha = 25°$
$\Delta T = 12°$ C

$\alpha = 20°$
$\Delta T = 17°$ C

FIGURE 1. Secondary flow patterns as seen from a head-on view of the plate surface.

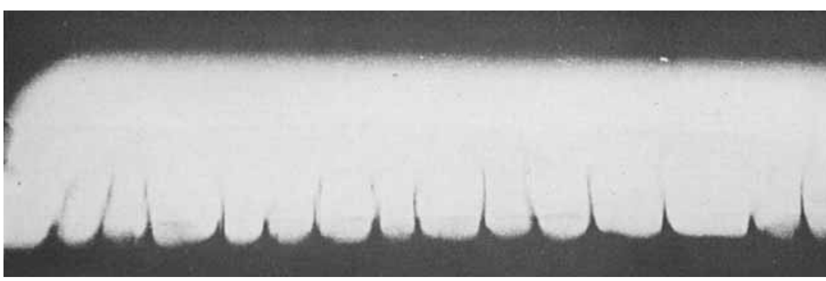
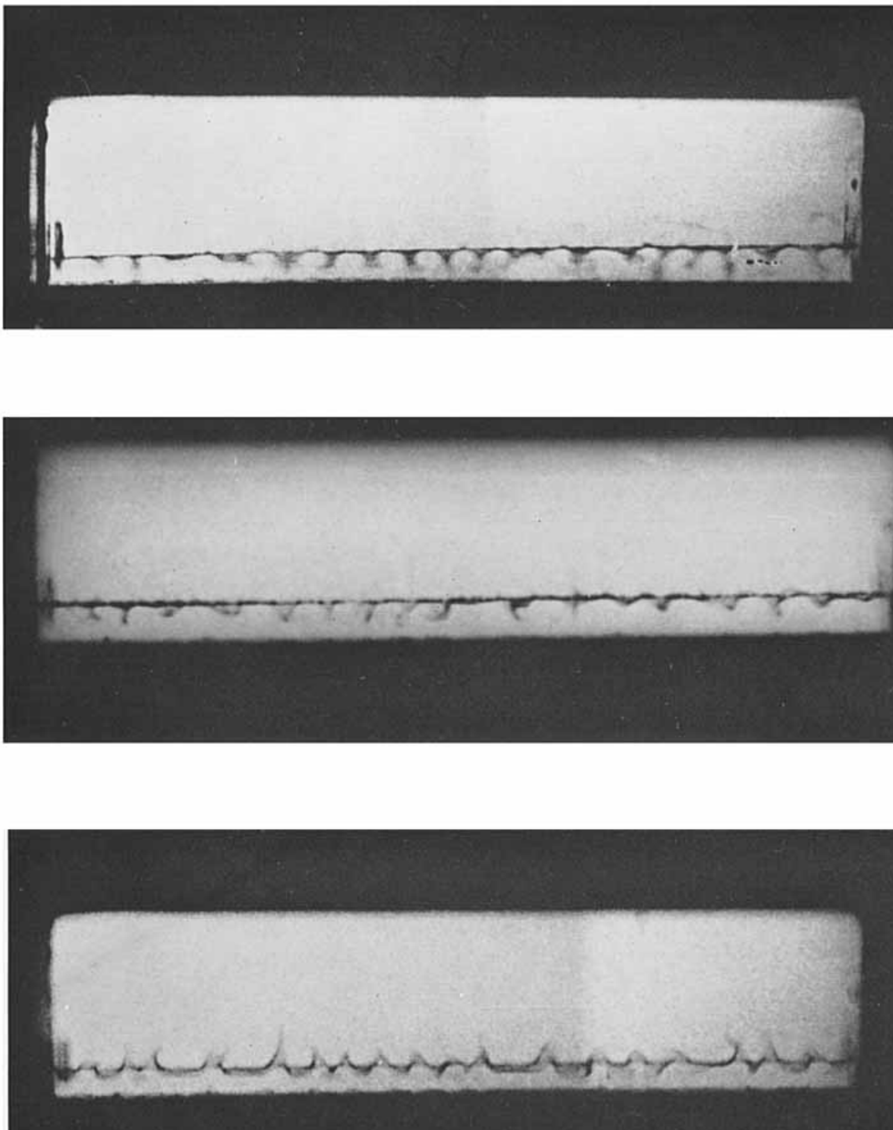


FIGURE 2. Secondary flow pattern as seen looking upward along the plate from the leading edge.



$x = 4{\cdot}3$ cm.

$x = 6{\cdot}5$ cm.

$x = 8{\cdot}0$ cm.

FIGURE 3. Observations of secondary flow in the boundary layer.